APPLICATION OF COMPLEX NUMBERS TO THE EXPLORATION OF QUADRILATERAL FLAT SHAPES

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ABSTRAK

Representasi bilangan kompleks sebagai titik-titik di bidang datar juga membantu dalam penyelesaian masalah matematika yang terkait dengan geometri, seperti soal-soal geometri di bidang, seperti syarat dua segmen garis sejajar, dan lain-lain. Tujuan dari penelitian ini adalah mengeskplorasi konsep analisis kompleks pada bangun datar segi empat. Kemudian, menemukan solusi permasalahan geometri menggunakan konsep bilangan kompleks. Penelitian ini menggunakan pendekatan kualitatif dengan fokus pada studi literature, yaitu melakukan tinjauan terhadap literatur yang ada mengenai penerapan bilangan kompleks dalam geometri bidang, khususnya pada bangun datar segiempat. Keabsahan data menggunakan teknik triangulasi untuk memeriksa kualitas data yang telah diperoleh dari berbagai sumber. Teknik analisis data mengikuti model Miles, Huberman, dan Saldana yaitu kondensasai data, display data, dan penarikan kesimpulan. Hasil dari penelitian ini menunjukkan bahwa terdapat sifat istimewa dari jajar genjang, penentuan hasil pencerminan suatu titik terhadap garis tertentu, penentuan letak titik tinggi dari suatu segitiga jika diketahui ketiga titik sudutnya dan dua tipe soal terakhir yang dibahas adalah soal OSAMO 2015 No. 2 dan OSN SMA 2009 tentang pembuktian dua segmen garis tegak lurus dan 4 titik tertentu membentuk suatu segiempat talibusur. Kontribusi penelitian ini memberikan pemahaman tentang hubungan antara bilangan kompleks dan geometri bidang, khususnya dalam konteks eksplorasi bangun datar segiempat. Serta mengembangkan pendekatan baru atau alternatif dalam menganalisis segiempat yang dapat memperluas wawasan dan keterampilan matematis siswa atau pembaca dalam memahami konsep tersebut.

Kata kunci: geometri, bilangan kompleks, segiempat, bangun datar

ABSTRACT

The representation of complex numbers as points in the plane also helps in solving mathematical problems related to geometry, such as geometry problems in the plane, such as the condition that two line segments are parallel, and others. The purpose of this research is to explore the concept of complex analysis on rectangular flat shapes. Then, finding solutions to geometry problems using the concept of complex numbers. This research uses a qualitative approach with a focus on literature studies, namely conducting a review of the existing literature on the application of complex numbers in plane geometry, especially in quadrilateral flat

shapes. Data validity uses triangulation techniques to check the quality of data that has been obtained from various sources. Data analysis techniques follow the model of Miles, Huberman, and Saldana, namely data condensation, data display, and conclusion drawing. The results of this study show that there are special properties of parallelograms, determination of the result of mirroring a point to a certain line, determination of the location of the high point of a triangle if the three angles are known and the last two types of questions discussed are OSAMO 2015 No. 2 and OSN SMA 2009 questions about proving two perpendicular line segments and 4 certain points form a quadrilateral talibusur. The contribution of this research is to provide an understanding of the relationship between complex numbers and plane geometry, especially in the context of exploring quadrilateral flat shapes. As well as developing new or alternative approaches in analyzing quadrilaterals that can broaden students' or readers' horizons and mathematical skills in understanding the concept.
Keywords: *geometry, complex numbers, quadrilaterals, flat shapes.*

INTRODUCTION

Flat plane geometry problems can usually be solved using existing definitions, axioms, and theorems. According to Chandra, flat plane geometry problems can also be solved using the analytic geometry approach. Although this approach is useful under the right conditions, it has some problems. According to Chen, some of the problems include: 1) Determining a point on a plane requires two variables. 2) The equations formed in problems involving geometric transformations in the plane are generally complex and therefore difficult to solve. Fortunately, these problems can be solved using complex numbers.

The representation of complex numbers as points on the plane naturally creates a two-way interaction between geometry and number. In this research article complex numbers are used to solve geometry problems in the plane namely: 1) the condition that two segments are parallel, 2) the condition that three points A, B, C are in line, 3) how to determine the midpoint of a line segment, 4) how to determine the diagonals of a rhombus are perpendicular to each other and 5) the condition that 4 points lie on one circle using crossratio. Meanwhile, in Shaw's research article, the use of complex numbers is used to solve problems about 1) the line connecting the midpoints of the sides of a triangle divides the triangle into 4 triangles of equal area and the triangle formed is an equilateral triangle and 2) the sufficient condition for a rectangle to be a parallelogram is that its diagonals intersect at equal length.

In relation to the previous background, the author is interested in using the complex number method to prove certain properties of rectangular geometry.

Complex number is a mathematical concept that allows us to combine real numbers with imaginary numbers. Complex numbers can be expressed in the form $a + bi$, where a and b are real numbers and i are imaginary units. Complex numbers can be used to calculate and analyze geometric properties, such as sides, angles, and volumes. In this study, complex numbers are used to solve flat geometry problems, such as the condition that two line segments are parallel, the condition that three points are in line, and how to determine the midpoint of a line segment. A, B, C line, and how to determine the midpoint of a line segment. Complex numbers are also used to solve problems related to complex numbers, such as determining the result of mirroring a point on a certain line and determining the location of the high point of a triangle if the three angles are known.

A binary complex number, also known as a conjugate, is a complex number that has special properties. The conjugate complex number of a complex number $z = x + iy$ is $\bar{z} = x - iy$. These conjugate complex numbers have several important properties that allow applications in various fields of mathematics and physics.

Quadrilateral exploration is a research that focuses on the use of complex numbers to understand and calculate the geometric properties of quadrilaterals. In this research, complex numbers are used as a way to analyze and calculate quadrilateral properties, such as sides, angles, and volume.

Complex numbers of the form $z = x + iy$ with x and y real numbers and $i = \sqrt{-1}$. Real numbers x is called the real part of z and is denoted by Re(z), $x = Re(z)$. While the real numbers y is called the imaginary part of z and is denoted by lm(z), $y = lm(z)$. Furthermore, the set of all complex numbers is denoted by $\mathbb{C}$. Thus, $\mathbb{C} = \{x + iy: x, y \ \in \mathbb{R}\}$ the set of all real numbers $\mathbb{R}$ can be summarized as the set of complex numbers z with $lm(z) = 0$. The set of all complex numbers z with $Re(z) = 0$ is denoted by $i\mathbb{R}$. Numbers $z \in iR$ are usually called pure imaginary numbers.

Algebraic operations of complex numbers on $\mathbb{C}$ are defined like algebraic operations on R by replacing $i^2 = -1$ if the term appears. Suppose $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$.

a) Summation: $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$
b) Deduction: $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$
c) Multiplication: $z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$$z_1 \cdot z_2 = x_1x_2 + x_1y_2i + x_2y_1i + y_1iy_2$$
$$z_1 \cdot z_2 = (x_1x_2 + y_1y_2i^2) + (x_1y_2i + x_2y_1i)$$
$$z_1 \cdot z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$$

d) Division: $\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2}$

$$\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2}\left(\frac{x_1-iy_1}{x_2-iy_2}\right)$$
$$\frac{z_1}{z_2} = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\,\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2},\ z_2 \neq 0$$

Modulus or absolute value of a complex number $z = x + iy$, written with the notation $|z|$is defined as: $|z| = \sqrt{x^2 + y^2}$ from the above definition, the relationship between $|z|, Re(z)$, and $lm(z)$, which is $|z|^2 = \left(Re(z)\right)^2 + \left(lm(z)\right)^2$. According to Spigel if $z_1, z_2, \dots z_m$ is a complex number, then the properties of complex numbers are as follows:

1) $|z_1z_2| = |z_1||z_2|$ or $|z_1z_2 \dots z_m| = |z_1||z_2| \dots |z_m|$
2) $\left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|$ if $z_2 \neq 0$

A conjugate complex number operation of the form $z = x + iy$ usually denoted $\bar{z}$ which is defined $\bar{z} = x - iy$. The algebraic operation of conjugate complex numbers in the set of $\mathbb{C}$ satisfies the following properties:

- For every complex number z holds
  1) $\bar{z} = z$
  2) $z + \bar{z} = 2Re(z)$
  3) $z - \bar{z} = 2i\ lm(z)$
  4) $z\bar{z} = \left(Re(z)\right)^2 + \left(lm(z)\right)^2 = |z|^2$

- If z_1, z_2 two arbitrary complex numbers, then
  1) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
  2) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
  3) $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
  4) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

The following two properties about whether a complex number is a real number or a pure imaginary number will be used frequently in the discussion in the next subchapters.

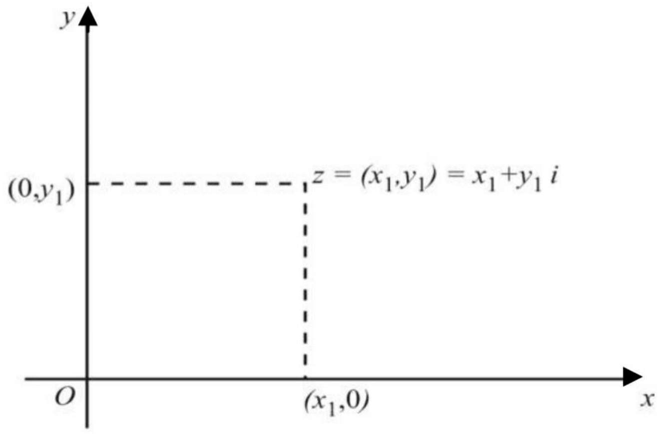
1. $z = \bar{z}$, if and only if $z \in \mathbb{R}$.
2. $z = -\bar{z}$, if and only if $z \in \mathbb{R}$.

DISCUSSION

A. Concept of complex numbers in geometry

Complex numbers are numbers that consist of real and imaginary numbers. A complex number $\mathrm{z} = (\mathrm{x}, \mathrm{y}) = \mathrm{x} + \mathrm{iy}$is geometrically expressed as a point (x, y) on the Cartesian plane. Thus, all complex numbers can be represented by all points on the Cartesian plane.

On the complex plane, the axes x as the real axis while the y as the imaginary axis. This cartesian plane is commonly called the complex plane or argand



plane.

Figure 1

x : Real axis

y : Imaginary axis

For example

$z = 1 + 2i$ represented with (1,2) as shown below:

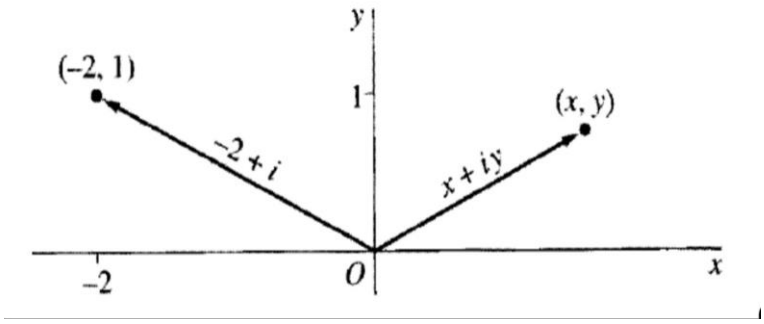


Figure 2

A complex number $z = x + iy$ can also be expressed as a vector in the complex plane with a base point (0,0) and an end point of x, y. In a limited sense a complex number $z = (x, y) = x + iy$ can be viewed as a vector (x, y) and the operations of addition and subtraction of two complex numbers are geometrically similar to those on a vector.

In accordance with the definition of the sum of two complex numbers $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$, the number $z_1 + z_2$ corresponds to the point whose coordinates $(x_1 + x_2, y_1 + y_2)$ or the vector with these components and $z_1 + z_2$ can be obtained vectorially as illustrated in the following figure

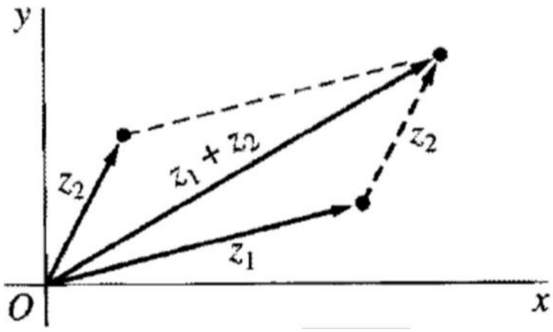


Figure 3

Subtraction $z_1 - z_2 = \text{z} + (-z_2)$ relates to the point whose coordinates $(x_1 - x_2, y_1 - y_2)$ or the vector with that component. $z_1 - z_2$ can also be obtained vectorially as illustrated in the following figure

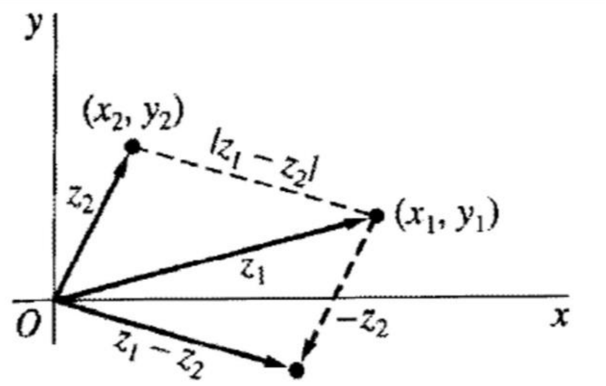


Figure 4

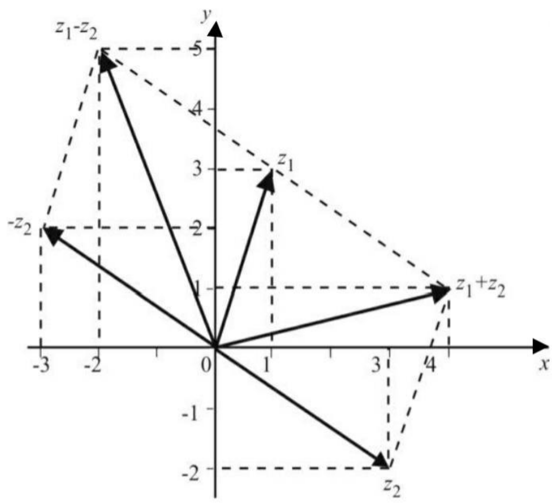
Example:

Given a complex number $z_1 = 1 + 3i$ and $z_2 = 3 + 2i$.

Describe the complex numbers $z_1, z_2, z_1 + z_2$, and $z_1 - z_2$ in the same way as vector addition and subtraction.

Completion:

Placed on one complex plane.



When calculated, the result of the above drawing must match

$z_1 + z_2$ = $(1 + 3i)$ + $(3 - 2i)$ = 4 + i

$z_1 - z_2$ = $(1 + 3i)$– $(3 - 2i)$ =– 2+$5i$

Although the multiplication of complex numbers z_1 and z_2 are complex numbers that can be expressed by vectors that lie in the same plane as z_1 and

z_2 but the product is not a scalar or vector product as commonly used in vector analysis.

Modulus or absolute value of a complex number $\mathrm{z} = (\mathrm{x}, \mathrm{y})$ is defined as $\sqrt{x^2 + y^2}$ and is denoted by $|z|$, i.e. $|z| = \sqrt{x^2 + y^2}$.

Geometrically $|z|$ is the distance between the origin and the point (x, y) or the length of the vector representing *z*. If *z is a* real number then $|z|$ = *z*. *The* inequality $z_1 < z_2$ means z_1 and z_2 are real numbers, $|z_1| < |z_2|$ means that the distance z_1 to the origin is closer than the distance z_2 to the origin.

Example:

Because $|-3 + 2i| = \sqrt{13}$ and $|1 + 4i| = \sqrt{17}$ then the point $-3 + 2i$ is closer to the origin than the point $1 + 4i$.

Distance between points $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ can be seen as the length of the vector representing $z_1 - z_2$(see figure 3), which is $|z_1 - z_2|$.

Because $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

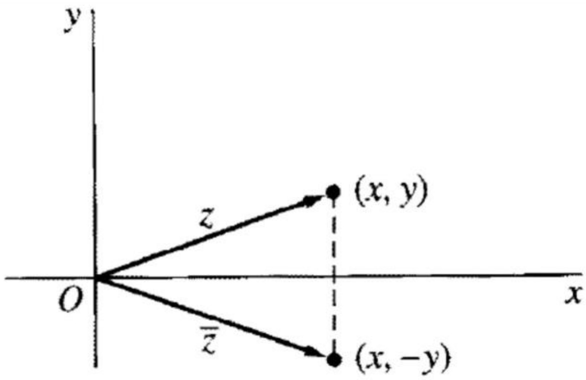
Then $|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

The conjugate of a complex number *z* = *x* +*iy is* defined as a complex number

z = *x*– *iy* and expressed $\bar{z}$, i.e.

$$\bar{z} = \mathrm{z} - \mathrm{iy}$$

In complex number geometry $\bar{z}$ is the point (*x*,– *y*) which is a reflection on the real axis of the point (*x*,*y*) representing *z*



Note that

$\bar{\bar{z}} = z$ and $|\bar{z}| = |z|$ for all *z*

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$then

$\overline{z_1 + z_2} = (x_1 + x_2) - i(y_1 + y_2) = (x_1 - iy_1) + (x_2 + iy_2)$

So the conjugate of the sum is

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

Furthermore, it can be shown

$\overline{z_1 + z_2} = \bar{z}_1 - \bar{z}_2$

$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{z_2} \qquad (z_2 \neq 0)$

The sum $z + \bar{z}$ of the complex numbers $z = x + iy$ and $z = x - iy$ is a real number *2x* and the subtraction of $z - \bar{z}$ is the pure imaginary number *2iy*. So that

$Re\, z = \frac{z + \bar{z}}{2}, \qquad Im\, z = \frac{z - \bar{z}}{2}$

An important identity associated with the conjugate is

$z\,\bar{z} = |z|^2$

This gives a way to determine the quotient $\frac{z_1}{z_2}$ by multiplying $\bar{z}_2$ on the numerator and denominator, so that the denominator becomes $|z_2|^2$.

Example:

For illustration

$$\frac{-1 + 3i}{2 - i} = \frac{(-1 + 3i)(2 + i)}{(2 - i)(2 + i)} = \frac{-5 + 5i}{|2 - i|^2} = \frac{-5 + 5i}{5} = -1 + \mathrm{i}$$

B. Application of Complex Numbers to explore the properties of quadrilaterals

The following presents problems on flat geometry that can be solved with complex numbers. Selected some problems that represent problems related to alignment, perpendicularity, alignment and quadrilateral talibus. The selected problems are considered easier to solve with the concepts and properties of complex numbers than using the laws of Euclid geometry.

Problem 1: This problem is an example of a problem taken from the article Geometric Aplication of Complex Number pp. 5.

Suppose $ABCD$ is a parallelogram and E, F, G, H is the midpoint of each line AB, BC, CD, DA. Show that $EG \parallel BC$ and $FH \parallel CD$ and $EFGH$ are parallelograms.

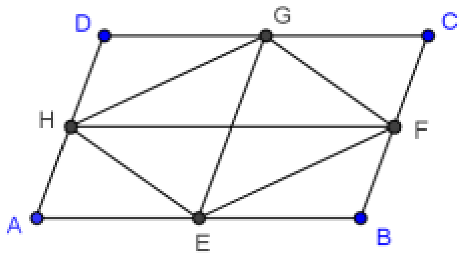


Figure 5. $ABCD$ is a parallelogram with E, F, G, H is the midpoint of each side.

Completion:

Suppose $a, b, c, dan\ d$ are consecutive complex numbers corresponding to points $A, B, C, dan\ D$, using property1, the complex numbers $\frac{1}{2}(a+b)$ represents the midpoint of E while the complex number $\frac{1}{2}(c+d)$ represents the midpoint of G. Thus, $EG = \frac{1}{2}(c+d-a-b)$.

Unknown: $ABCD$ is a parallelogram, then $b-a=c-d$ and $d-a=c-b$. So

$$EG = \frac{1}{2}(c+d-a-b)$$
$$= \frac{1}{2}(d-a+c-b)$$
$$= \frac{1}{2}(c-b+c-b) = c-b = BC$$

So, EG and BC are parallel and equal in length. Further, analogous to the above method is obtained

$$HF = \frac{1}{2}(b+c-a-d)$$
$$= \frac{1}{2}(b-a+c-d)$$
$$= \frac{1}{2}(c-d+c-d) = c-d = DC$$

So, HF and DC are parallel and equal in length.

<u>Problem 2:</u> This problem is problem no. 1.8 of M. R. Spigel's book, Complex Variables pp. 24.

Prove that the two diagonals in a parallelogram intersect each other at the center.

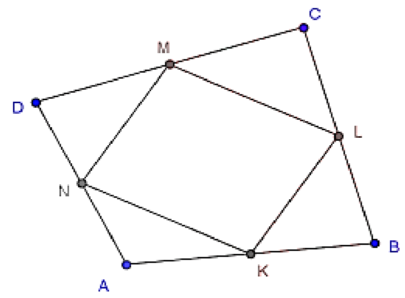


Figure 6. The two diagonals in a parallelogram intersect each other at thecenter.

Completion:

Take a look at the figure 6. Suppose $OABC$ is a parallelogram with diagonals intersecting at P. suppose also $OA = z_1$ and $OC = z_2$. Thus, $OB = z_1 + z_2$.

Because $z_1 + AC = z_2$, $AC = z_2 - z_1$then $AP = m(z_2 - z_1)$ with $0 \leq m$. In the same way obtained $OP = n(z_2 + z_1)$ with $0 \leq n \leq 1$. Then it will be shown $m = n = \frac{1}{2}$.

However $OA + AP = OP$ is a pure imaginary number, so $z_1 + m(z_2 - z_1) = n(z_2 + z_1)$ or $(1 - m - n)\, z_1 + (m - n)\, z_2 = 0$. Thus, according to property 2:

$(1 - m - n) = 0$ and $m - n = 0$ means $m = \frac{1}{2}, n = \frac{1}{2}$.

So it is proven that P is the midpoint of the diagonal.

Problem 3: Proof of Varignon's theorem.

The midpoints of the sides of an arbitrary quadrilateral form a parallelogram.

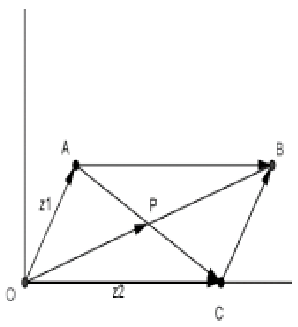


Figure 7. *Varignon's Theorem*

Completion:

Suppose $a, b, c, d, k, l, m,$ and n are the complex numbers corresponding to the points A, B, C, D, K, L, M and N. Given the midpoints $K, L, M,$ and N, as shown. Using property 1, the complex number $k = \frac{1}{2}(a+b)$ represents the midpoint of the line AB. In the same way to show the results $l, m,$ and n. Therefore

$$l - k = \frac{1}{2}(b+c) - \frac{1}{2}(a+b)$$

$$= \frac{1}{2}(c-a) \quad \text{And}$$

$$m - n = \frac{1}{2}(c+d) - \frac{1}{2}(d+a)$$

$$= \frac{1}{2}(c-a)$$

So that $l - k = m - n$, so the line $KL = MN$ therefore $KLMN$ is a parallelogram.

Problem 4: Proof of Van Aubel's theorem.

If on a quadrilateral to the right of the long side of the quadrilateral is placed a square that corresponds to the length of the side of the quadrilateral, then the two lines connected to the midpoint of the center of the square are equal in length and perpendicular.

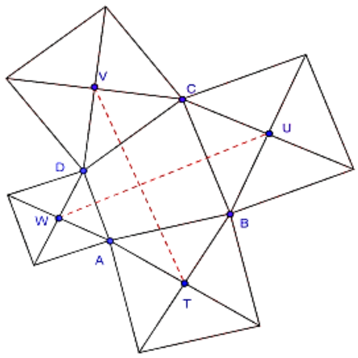


Figure 8. *Van Aubel's Theorem*

Completion:

Suppose a, b, c, d, t, u, v and w are the complex numbers corresponding to points A, B, C, D, T, U, V and W. Consider the quadrilateral $ABCD$ and the center points of the square $T, U, V, W,$ as shown in the figure 8. Next, the three points A, T, B form the *vertex of* a square. Thus, using property 1 is obtained:

$$t = \frac{1}{2}(a + b) + \frac{1}{2}i(a - b)$$

In the same way $u = \frac{1}{2}(b + c) + \frac{1}{2}i(b - c)$

$$v = \frac{1}{2}(c + d) + \frac{1}{2}i(c - d)$$

$$w = \frac{1}{2}(d + a) + \frac{1}{2}i(d - a)$$

So $t - v = \frac{1}{2}(a + b - c - d) + \frac{1}{2}i(a - b - c + d)$ and

$$u - w = \frac{1}{2}(-a + b + c - d) + \frac{1}{2}i(a + b - c - d) = i(t - v)$$

Proven UW and TV are perpendicular and equal in length.

<u>Problem 5:</u> Proof of *Thebault's first* theorem

If on each side of a parallelogram a square is formed then the center of the four squares will be a square.

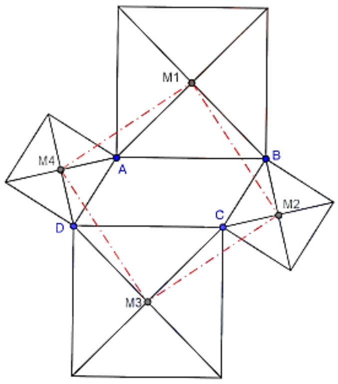


Figure 9. *Thebault's first theorem*

Completion:

Suppose p and q denote the complex numbers corresponding to DC and CB. Suppose also $M_1, M_2, M_3, dan\ M_4$ is a point of the square as shown in fig.5. So

$DE + EM_3 = DM_3 = \frac{p}{2} - \frac{p}{2}i$

Analog acquired:

$CM_3 = -\frac{p}{2} - \frac{p}{2}i$

$CM_2 = \frac{q}{2} - \frac{q}{2}i$

$BM_2 = -\frac{q}{2} - \frac{q}{2}i$

$AM_1 = \frac{p}{2} + \frac{p}{2}i$

$BM_1 = \frac{-p}{2} + \frac{p}{2}i$

From the above results can be obtained

V $M_3M_2 = \frac{p+q}{2} + \frac{(p-q)i}{2}$

C $M_2M_1 = \frac{-p+q}{2} + \frac{(p+q)i}{2}$ Thus,

$(M_3M_2)i = M_2M_1; M_3M_2 = M_2M_1, M_3M_2 \perp M_2M_1$

In the same way, it can be **shown** $(M_1M_4)i = M_4M_3; M_1M_4 = M_4M_3, M_3M_2 \perp M_2M_1$.

Problem 6: The following will prove that in a rhombus the diagonals are perpendicular to each other.

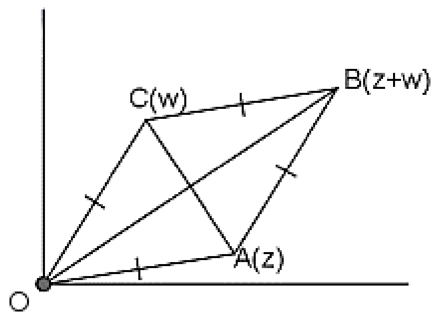


Figure 10. The diagonals of a rhombus are perpendicular to each other.

Completion:

Suppose $OABC$ is a rhombus with $OA = z$ and $OC = w$. This means $|z| = |w|$ and $OB = z + w$. So, $AC = w - z$. It will be shown $OB \perp AC$. Using property 3 it is shown that:

$\frac{w-z}{z+w-o} \in iR \leftrightarrow \frac{w-z}{z+w} + \left(\overline{\frac{w-z}{z+w}}\right) = 0$

$\frac{w-z}{z+w} + \left(\overline{\frac{w-z}{z+w}}\right) = \frac{(\bar{z}+\bar{w})(w-z)(z+w)(\bar{w}-\bar{z})}{|z+w|^2}$

$= \frac{\bar{z}w - z\bar{w} + z\bar{w} - \bar{z}w}{|z+w|^2} = 0$ proven.

Problem 7: If each diagonal of a quadrilateral is bisected, a parallelogram is formed.

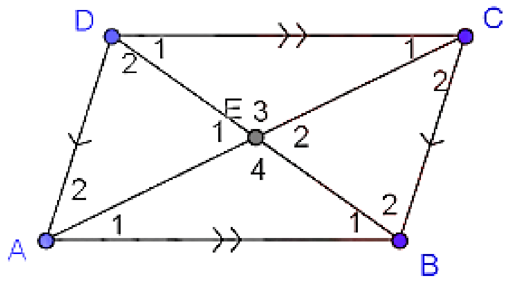


Figure 11. The diagonals of a parallelogram divide in half at equal length.

Completion:

Suppose $A(z_1), B(z_2), C(z_3), D(z_4)$ is a corner point of a quadrilateral whose two diagonals intersect at the center. And suppose also E is the intersection point of AC and BD.

Find out $BE = DE$ and $AE = CE$. Since E is the midpoint of AC and BD, using property 1, we get:

$z = \frac{1}{2}(z_1 + z_3)$ and $z = \frac{1}{2}(z_2 + z_4)$. This results in;

$\frac{1}{2}(z_1 + z_3) = \frac{1}{2}(z_2 + z_4)$ or $(z_2 - z_1) = (z_3 - z_4)$

This means AD and BC are equal in length and parallel. Analogs are obtained AD and BC are equal in length and parallel. From these evidences, it can be seen that the figure above is a parallelogram.

Problem 8: Given three points A, B and C that are not inline.

Suppose z is the mirroring of C on the line AB. Then z can be expressed in the following form:

$$z = \frac{a\bar{c} + b\bar{a} - a\bar{b} - b\bar{c}}{\bar{a} - \bar{b}}$$

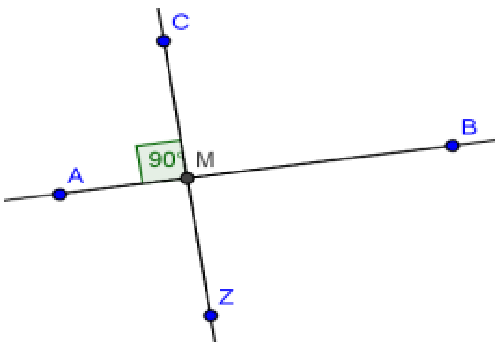


Figure 12. z is the result of mirroring the point C on the line AB

Completion:

Suppose a, b, c and z are the complex numbers corresponding to the points $A, B, C,$ and Z. Suppose also M is the midpoint of ZC. Thus,

$$m = \frac{z+c}{2}$$

Since z is the result of mirroring C with respect to line AB, M must lie on line AB. Hence, according to property 3:

$$\frac{m-b}{a-b} = \overline{\left(\frac{m-b}{a-b}\right)}$$

Becaus e $m = \frac{z+c}{2}$, then

$$\frac{z+c-2b}{2(a-b)} = \frac{\bar{z}+\bar{c}-2\bar{b}}{2(\bar{a}-\bar{b})}$$

Because ZC perpendicular to the line AB then,

$$\frac{z-c}{a-b} = \left(\frac{\overline{z-c}}{a-b}\right) = \frac{\bar{c}-\bar{z}}{\bar{a}-\bar{b}}$$

So that the equation is obtained

$$\frac{z+c-2b}{2(a-b)} = \frac{\bar{z}+\bar{c}-2\bar{b}}{2(\bar{a}-\bar{b})} \qquad (1)$$

$$\frac{z-c}{a-b} = \frac{\bar{c}-\bar{z}}{\bar{a}-\bar{b}} \qquad (2)$$

Both equations (1) and (2) can be thought of as a system of equations in z and $\bar{z}$. The system of equations z and $\bar{z}$ can be presented in the following form:

$Pz + Q\bar{z} = R$

$Uz + V\bar{z} = W$ (3)

By

$P = \bar{a} - \bar{b} = U$

$Q = a - b$

$V = -(a-b) = b - a$

$R = a\bar{c} - b\bar{c} + \bar{a}c - \bar{b}c$

$W = a\bar{c} - b\bar{c} - 2a\bar{b} - \bar{a}c + 2a\bar{b} + \bar{b}c$ (4)

If the system of equations (3) and (4) is solved by the system of equations for z then obtained

$$z = \frac{\begin{vmatrix} R & Q \\ W & V \end{vmatrix}}{\begin{vmatrix} P & Q \\ U & V \end{vmatrix}} = \frac{RV-QW}{PV-QU} = \frac{a\bar{c}+b\bar{a}-a\bar{b}-\bar{b}c(2b-2a)}{(\bar{a}-\bar{b})(2b-2a)}$$

$$= \frac{a\bar{c}+b\bar{a}-a\bar{b}-\bar{b}c}{\bar{a}-\bar{b}}$$

CONCLUSION

Complex numbers are numbers that consist of real and imaginary numbers. A complex number $\mathrm{z} = (\mathrm{x}, \mathrm{y}) = \mathrm{x} + \mathrm{i}\mathrm{y}$is geometrically expressed as a point (x, y) on the Cartesian plane. Thus, all complex numbers can be represented by all points on the Cartesian plane. In the complex plane, the axes x as the real axis while the y as the imaginary axis.

This paper has discussed how to prove and solve some flat geometry problems that can be solved with the help of complex numbers. By inserting the formulas of the properties of complex numbers related to flat geometry problems can be solved. The types of problems solved are proving two special properties of parallelogram, determining the result of mirroring a point on a certain line. The last two types of problems discussed are OSAMO 2015 problem no.2 and OSN 2009 about proving two perpendicular line segments and four given points form a quadrilateral talibus. Readers are advised to use complex numbers if they encounter difficulties in trying to solve geometry problems related to parallelism, perpendicularity of two line segments and problems involving collinearity and concyclicity. This study does not compare the complex number approach with the classical method but only as a complement to the existing proof methods.

The use of complex numbers in solving geometry problems on quadrilateral flat figures provides a deeper understanding of the relationship between complex numbers and plane geometry. This approach also makes it possible to identify geometric properties of quadrilaterals more easily and efficiently than using traditional Euclid geometry methods. In addition, the proof of some geometry theorems such as Varignon's theorem, Van Aubel's theorem, and Thebault's first theorem can also be done using complex numbers. Mathematical operations on complex numbers such as modulus, conjugate, and distance between two complex numbers can also be applied in the context of flat geometry.

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