STRATEGIES FOR APPLYING COMPLEX ANALYSIS IN SOLVING ALGEBRA PROBLEMS

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ABSTRAK

Analisis kompleks memegang peranan penting dalam aljabar modern, terutama dalam menyelesaikan persamaan-persamaan aljabar yang tidak dapat diselesaikan dengan bilangan real. Namun, penerapan analisis kompleks ini membutuhkan strategi khusus agar dapat diimplementasikan secara efektif. Penelitian ini bertujuan untuk menganalisis dan merumuskan strategi penerapan analisis kompleks dalam menyelesaikan masalah aljabar serta memberikan contoh penerapan strategi tersebut pada kasus-kasus nyata. Pendekatan kualitatif dengan jenis penelitian studi literatur dan analisis konten digunakan dalam penelitian ini. Sumber data berasal dari jurnal, buku teks, dan referensi tertulis lainnya terkait analisis kompleks dan aljabar. Teknik pengumpulan data dilakukan dengan studi dokumentasi. Keabsahan data diperoleh melalui triangulasi sumber. Analisis data menggunakan model Miles, Huberman, dan Saldana dengan tahapan reduksi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian ini berupa strategi komprehensif penerapan analisis kompleks dalam menyelesaikan masalah aljabar yang mencakup tahapan identifikasi konsep, metode analisis kompleks yang relevan, masalah aljabar yang akan diselesaikan, serta contoh penerapan strategi secara rinci dan sistematis. Strategi yang dirumuskan dalam penelitian ini dapat digunakan sebagai panduan dalam mengaplikasikan analisis kompleks untuk menyelesaikan berbagai permasalahan aljabar secara efektif, baik dalam bidang akademik maupun praktis.

Kata Kunci: Analisis Kompleks, Aljabar, Strategi Penerapan, Persamaan Aljabar

ABSTRACT

Complex analysis plays an important role in modern algebra, especially in solving algebraic equations that cannot be solved with real numbers. However, the application of complex analysis requires special strategies to be implemented effectively. This study aims to analyze and formulate strategies for applying complex analysis in solving algebraic problems and provide examples of the application of

these strategies in real cases. A qualitative approach with the research type of literature study and content analysis is used in this research. Data sources come from journals, textbooks, and other written references related to complex analysis and algebra. The data collection technique was done by documentation study. Data validity was obtained through source triangulation. Data analysis used the Miles, Huberman, and Saldana model with the stages of data reduction, data presentation, and conclusion drawing. The results of this study are in the form of a comprehensive strategy for applying complex analysis in solving algebraic problems which includes the stages of concept identification, relevant complex analysis methods, algebraic problems to be solved, and examples of strategy application in detail and systematically. The strategy formulated in this study can be used as a guide in applying complex analysis to solve various algebraic problems effectively, both in academic and practical fields.
Keywords: *Complex Analysis, Algebra, Application Strategies, Algebraic Equations*

INTRODUCTION

Algebra is a branch of mathematics that plays an important role in various fields of science, such as physics, engineering, economics, and computing. Purwosetiyono stated that algebra is a branch of mathematics that has broad applications in various other fields of science because it studies the concepts underlying the mathematical structure (Purwosetiyono, 2022). In algebra, there are often equations that cannot be solved using only real numbers. Dwi Tristianto explained that there are many algebraic equations that cannot be solved using only real numbers, such as polynomial equations with degrees greater than four (Dwi Tristianto, 2018). In these situations, complex analysis offers solutions by utilizing complex numbers and the properties of analytic functions. Kadir mentioned that complex analysis studies the properties of complex variable functions and has an important role in solving algebraic equations involving complex numbers (Kadir, 2016).

Marsitin defines complex analysis as a branch of science that specializes in studying functions with complex variables and their properties (Marsitin, 2017). Zetriuslita explains that concepts such as analytic functions, Taylor series, contour integrals, and residue theory are key concepts in complex analysis that have important applications in modern algebra (Zetriuslita, 2014). By using the tools of complex analysis, difficult algebraic equations can be simplified, complicated integrals can be calculated, and the properties of algebraic functions can be better analyzed. Kusumawinahyu states that complex analysis provides useful tools to simplify algebraic equations, calculate complicated integrals, and analyze the properties of algebraic functions (Kusumawinahyu, 2017).

Sari asserts that complex analysis offers effective tools and methods for solving algebraic equations that cannot be solved with real numbers (Sari, 2014). Nonetheless, the application of complex analysis in solving algebraic problems requires specific strategies in order to be implemented effectively. A deep understanding of the concepts of complex analysis and the ability to integrate them with algebraic principles is very important. Purnama and Tanjungsari emphasized the importance of a strong conceptual understanding in complex analysis and the ability to integrate it with algebraic principles in order to effectively implement it (Purnama & Tanjungsari, 2018).

This study aims to analyze and formulate strategies for applying complex analysis in solving algebraic problems. With a systematic and comprehensive strategy, it is expected that complex analysis can be optimally utilized to solve complex algebraic problems, both in academic and practical fields.

This research uses a qualitative approach with literature study and content analysis. Data sources come from journals, textbooks, and other written references related to complex analysis and algebra. The data collection technique was carried out by documentation study, namely collecting, reviewing, and analyzing various literature sources relevant to the research topic. Data validity was obtained through source triangulation, which is comparing and confirming information from various sources to ensure the accuracy and validity of the data. Data analysis used the Miles, Huberman, and Saldana (2014) model with the stages of data reduction, data presentation, and conclusion drawing.

DISCUSSION

A comprehensive strategy for applying complex analysis in solving algebraic problems involves several important stages that must be followed systematically. The first stage is the identification of basic concepts of complex analysis relevant to the algebraic problem to be solved.

A. Basic Concepts of Complex Analysis

Before applying complex analysis methods, an in-depth understanding of the basic concepts of complex analysis is necessary. These concepts include complex numbers, analytic functions, complex integrals, Taylor series, and some other key principles.

1. Complex Numbers

   Complex numbers are the main foundation in complex analysis. Complex numbers can be expressed in cartesian form $(a + bi)$ or polar form $(re^{i\theta})$, where a dan b are real numbers, r are magnitudes, and θ is the argument (Marsitin, 2017). Basic operations such as addition, subtraction, multiplication, and division apply to complex numbers, with slightly different rules from real numbers.

2. Analytic Function

   Analytic functions are a core concept in complex analysis. A function $f(z)$ is said to be analytic in a region if its complex derivative, $f'(z)$, exists at every point in the region (Aryani, 2014). Analytic functions have special properties such as satisfying the Cauchy-Riemann condition and can be expanded into a Taylor series around every point in the analytic region (Zetriuslita, 2014).

3. Complex Integral

   Complex integrals are one of the main tools in complex analysis. A complex integral is defined as the integral of a complex function $f(z)$ d along a closed path C in the complex plane (Gamelin, 2001). Complex integrals have many important applications, such as in residue theory and conformal transformations (Sari, 2014).

4. Taylor Series

   A Taylor series is an infinite series representation of an analytic function around a particular point in its analytic region (Munir, 2015). The Taylor series allows an analytic approach to solving differential equations and complex algebraic equations by using the series expansion (Kusumawinahyu, 2017).

5. Other Key Principles

   In addition to the above concepts, there are several other key principles in complex analysis that need to be understood, such as the Cauchy integral theorem, the residue theorem, and the argument principle (Kadir, 2016). A deep understanding of these basic concepts will make it easier to choose the right complex analysis method to solve a particular algebraic problem.

After identifying relevant complex analysis concepts, the next step is to understand the relationship between these concepts and the algebraic problem to be solved.

B. Identify the Algebra Problem to be Solved

The second stage in this strategy is to identify algebraic problems that will be solved using complex analysis. Some types of algebra problems that can be solved with complex analysis include:

1. Algebraic Equations that cannot be Solved with Real Numbers

   There are many algebraic equations that do not have solutions in real numbers, such as polynomial equations with more than four degrees. In such cases, complex analysis can be used to find solutions in complex numbers (Dwi Tristianto, 2018).

2. Integrals that are Difficult to Calculate Directly

   Some algebraic integrals that are difficult to calculate directly can be solved by using contour integrals in complex analysis. By choosing the right integration path in the complex plane, we can utilize the properties of analytic functions to compute such integrals (Kadir, 2016).

3. Analysis of Properties of Algebraic Functions

   Complex analysis can provide new insights into the properties of algebraic functions, such as finding points of singularity, determining residues, and analyzing the behavior of functions around critical points. By understanding these properties, we can obtain more effective solutions to algebraic problems involving such functions (Purwosetiyono, 2022).

4. Algebra Problems in Other Fields

   Complex analysis can also be used to solve algebraic problems that arise in other fields such as physics, engineering, economics, and computing. For example, in quantum physics, complex analysis is used to solve the Schrödinger equation (Zill & Shanahan, A First Course in Complex Analysis With Application (4th ed.), 2018)

After identifying the algebraic problem to be solved, the next step is to select and apply the appropriate complex analysis method to solve the problem.

C. Application of Complex Analysis Methods

The third stage in this strategy is to apply the appropriate complex analysis method to solve the identified algebraic problem. Here are some methods that can be used:

1. Complex Number Method

   If the algebraic problem at hand involves equations that cannot be solved with real numbers, then the complex number method can be used. In this method, we allow solutions in complex numbers and use the properties of complex numbers to find the solution of the equation (Zill & Shanahan, A First Course in Complex Analysis With Application (4th ed.), 2018).

   Example: An algebraic equation $x^4 + 1 = 0$ has no solution in real numbers. However, using the complex number method, we can find solutions in the form: $x = \pm 1 , \pm i$

2. Taylor and Laurent Series Method

   If the algebraic problem involves complex functions, then Taylor and Laurent series methods can be used to represent the function around a point. By using the series representation, we can manipulate the function algebraically to obtain a simpler solution (Ahlfors, 2017).

   Example: Suppose we want to solve an algebraic equation: $(1+z)^4 = 2z.$ By using the Taylor series for $(1+z)^4$ di sekitar $z = 0$, we can simplify the equation to: $1 + 4z + 6z^2 + 4z^3 + z^4 = 2z.$ Then, by equalizing the coefficients z^n on both sides of the equation, we can find the solution of the equation.

3. Contour Integral Method

   If the algebraic problem involves integrals that are difficult to calculate directly, then the contour integral method can be used. In this method, we choose an appropriate integration path in the complex plane and utilize the properties of analytic functions to compute the integral (Bak & Newman, 2017).

   Example: Suppose we want to calculate the integral: $\int (z^2 + 1)^{(-1)}\, dz$. This integral is difficult to calculate directly. However, by using the contour integral along the circle $|z| = R$ in the complex plane, we can calculate the integral as:

$$\int (z^2 + 1)^{(-1)}\, dz = \left(\frac{1}{2\pi i}\right) \oint (z^2 + 1)^{(-1)}\, dz$$

   By utilizing the residue theory, we can calculate this contour integral and obtain the desired integral value.

4. Method of Analyzing the Properties of Algebraic Functions

   If the algebraic problem involves analyzing the properties of algebraic functions, then complex analysis methods can be used to determine the singularity points, residues, and the behavior of the function around critical points. By understanding these properties, we can obtain more effective solutions to algebraic problems involving such functions.

   Example: Suppose we want to analyze the properties of algebraic functions: $f(z) = \frac{z^3}{(z^2-1)}$

   Using the Laurent series around the singularity points $z = \pm 1$, we can determine the residues of the function at these points. This information can help us understand the behavior of the function around critical points and open up opportunities to solve problems involving the function more effectively.

   In applying these complex analysis methods, we need to pay attention to several important things, such as choosing the method that best suits the problem at hand, understanding the assumptions and limitations of each method, and ensuring the accuracy of the calculations and algebraic manipulations performed.

D. Example of Application of Complex Analysis Strategy in Algebra

One example of a frequently encountered algebraic problem is a polynomial equation of high degree, which is difficult to solve by conventional methods such as factorization or the use of the quadratic formula. Suppose we have an equation:

$$x^8 - 3x^6 + 2x^4 - x^2 + 1 = 0$$

Using complex analysis, we can transform this equation into the domain of complex numbers and apply a fundamental theorem of algebra. This theorem states that every polynomial of degree n d with complex coefficients has at most n complex roots.

The first step is to write the polynomial in the form of a product of linear factors:

$$(x - z_1)(x - z_2)(x - z_3) \dots (x - z_8) = 0$$

By using numerical methods such as Newton's iteration or Laguerre's method, we can find the complex roots $(z_1, z_2, z_3, \dots, z_8)$ of the polynomial. Once the complex roots are obtained, we can factorize the polynomial into linear factor product form and then solve the equation directly.

This approach is very useful in solving high-degree polynomial equations that are difficult to solve with conventional methods. However, it also has limitations in terms of numerical stability and computational efficiency, especially for polynomials of very high degree.

In complex analysis, complex numbers are expressed by letters z, while the letters x and y denote real numbers. If $z = x + iy$ denotes any complex number, then from the equation x denotes a real number and y denotes an imaginary number of z.

Each complex number represents a point in the plane and vice versa. Next, we will present some properties of complex numbers that will be used for the discussion of algebraic problems as follows:

1. $z_1 + z_2 \in \mathbb{C}\ and\ z_1 \cdot z_2 \in \mathbb{C}$ (closed nature)
2. $z_1 + z_2 = z_2 + z_1\ and\ z_1 \cdot z_2 = z_2 \cdot z_1$ (commutative property)
3. $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ and $(z_1 \cdot z_2) \cdot z_3 = z_1 \cdot (z_2 \cdot z_3)$ (associative property)

4. $z_1 \cdot (z_2 + z_3) = z_1 \cdot z_2 + z_1 \cdot z_3$ (distributive property)

The following presents problems about algebra that can be solved with complex numbers. The selected problems are considered easier to solve with the concepts and properties of complex numbers than using other solutions.

Problem 1

Find the real numbers x and y that satisfy $\frac{x-3}{3+i} + \frac{y-3}{3-i} = 1$

Completion:

Simplify the equation by equalizing the denominator to be

$\frac{(x-3)(3-i)}{(3+i)(3-i)} + \frac{(y-3)(3+i)}{(3-i)(3+i)} = 1$ thus obtained

$\frac{3x-9-xi+3i}{9-(-1)} + \frac{3y-9+yi-3i}{9-(-1)} = 1$, (remember $i^2 = -1$)

$\frac{3x-9-xi+3i}{10} + \frac{3y-9+yi-3i}{10} = 1$, both segments are multiplied by 10 to get

$3x - 9 - xi + 3i + 3x - 9 - xi + 3i = 10$

Grouping that has variables x, y, i and constants

$3x + 3y - xi + 3i + yi - 3i = 10 + 9 + 9$

$3x + 3y - xi + yi = 28$ or can be written $(3x + 3y) + (-x + y)i = 28 + 0i$

In general $a_1 + b_1 i = a_2 + b_2 i$ if $a_1 = a_2$ and $b_1 = b_2$ then obtained

- $3x + 3y = 28$ ……………. (1)
- $-x + y = 0$ ………………. (2)

Based on equation (2) then obtained $y = x$, substituted into equation (1) obtained

$3x + 3y = 28$

$3x + 3x = 28$ so $6x = 28 \rightarrow x = 4\frac{2}{3}$.

Because $y = x$ then $x = 4\frac{2}{3}$ and $y = 4\frac{2}{3}$.

Problem 2

If $z_1 = \left(2, \frac{1}{2}\right) = 2 + \frac{1}{2}i$ and $z_2 = \left(-3, \sqrt{2}\right) = -3 + \sqrt{2}i$, determine $2z_1$, $z_1 + 3z_2$ dan $2z_1 - z_2$

Completion:

- How much is $2z_1$

Let c be a scalar and $z = x + yi$ then $cz = cx + cyi$, for $z_1 = 2 + \frac{1}{2}i$

Obtained $2z_1 = 2(2) + 2\left(\frac{1}{2}\right)i$

$= 4 + i$

So $2z_1 = 4 + i$

- How much $z_1 + 3z_2$

  $3z_2 = 3(3) + 3\sqrt{2}i$

  $z_1 = x_1 + y_1 i \; and \; z_2 = x_2 + y_2 i \; then \; z_1 + z_2 = (x_1 + y_2) + (x_1 + y_2)$

  Then $z_1 + 3z_2 = (2 + (-9)) + \left(\frac{1}{2} + 3\sqrt{2}\right)i$

  $= (2 - 9) + \left(\frac{1}{2} + 3\sqrt{2}\right)i$

  $= -7 + \left(\frac{1}{2} + 3\sqrt{2}\right)i$

  So $z_1 + 3z_2 = -7 + \left(\frac{1}{2} + 3\sqrt{2}\right)i$

- How much $2z_1 - z_2$

  $z_1 = x_1 + y_1 i \; and \; z_2 = x_2 + y_2 i \; then \; z_1 - z_2 = (x_1 - y_2) + (x_1 - y_2)$

  Then $2z_1 - z_2 = \left(4 - (-3)\right) + \left(1 - \sqrt{2}\right)i$

  $= (4 + 3) + \left(1 - \sqrt{2}\right)i$

  $= 7 + \left(1 - \sqrt{2}\right)i$

  So $2z_1 - z_2 = 7 + \left(1 - \sqrt{2}\right)i$

Problem 3

Find the real numbers *x* and *y* that satisfy $(1 + 2i)x + (1 - 2i)y = 1 - i$

Completion:

Since the above problem is an equation of two complex numbers, keep in mind that $z_1 = a_1 + b_1 i \;\; and \;\; z_{2=} a_2 + b_2 i \;\; same, that \; is \; z_1 = z_2 \; if \; a_1 = a_2 \; and \; b_1 = b_2.$

Next, change the problem into the general form of addition of complex numbers $C\,(a_1 + b_1 i) = C\,a_1 + C\,b_1 i$ so that it becomes:

$x\,(1 + 2i) + y\,(1 - 2i) = 1 - i$

$x.1 + x.2i + y.i - y.2i = 1 - i$

$x + 2xi + y - 2yi = 1 - i$

Then categorized the variables into:

$x + y + (2x - 2y)i = 1 - i$

Based on the nature of the sum of complex numbers, it can be concluded that $x + y = a_1, (2x - 2y)i = b_1, 1 = a_2, and\ i = b_2$, thus the equation is obtained

- $a_1 = a_2 \rightarrow x + y = 1$ …………………..(1)
- $b_1 = b_2 \rightarrow (2x - 2y)i = -1$, both segments are divided by 2 into:

  $b_1 = b_2 \rightarrow x - y = -\frac{1}{2}$ ……………….(2)

Then from equations (1) and (2) are eliminated so that it becomes

$x + y = 1$

$x - y = -\frac{1}{2}$ +

$2x = \frac{1}{2}$, then $x = \frac{1}{4}$.

Then substitute $x = \frac{1}{4}$ into (1)

$x + y = 1$ or can be written $y = 1 - x$

$y = 1 - \frac{1}{4}$

$y = \frac{3}{4}$.

So the values that satisfy are $x = \frac{1}{4}$ and $y = \frac{3}{4}$.

The application of complex analysis in other algebra includes LRC circuits, complex numbers in RLC circuits are applied during calculations on the circuit. One calculation that uses complex numbers is impedance. In the Basic Theory section, it has been explained that impedance is the whole of the nature of resistance. It has also been explained the equation to find the amount of impedance. However, to find the actual impedance using complex numbers, with the equation :

$$z = R + jX_l + jX_c$$

$$z = Ze^{i\theta}$$

To find out whether the current or voltage vibrates first, ohm's law can be used:

$$I = \frac{V}{z} = \frac{V_0}{Z} e^{j(\theta - \emptyset)}$$

Which indicates the current is out of phase by ϕ from the voltage.

In solving RLC circuit problems, we have to transform complex numbers to perform addition, subtraction, multiplication, and division operations. Therefore, we need the ability to transform rectangular complex numbers to polar and vice versa. For every addition and subtraction operation, rectangular form should be used. While multiplication and division operations, polar form is used (Ghassani, 2015).

Example:

A load in an electrical circuit has an impedance value of tang written as a complex number $Z = (5 + j4)\Omega$, what is the impedance value when written in polar form?

Completion:

- Calculating the real price modulus value of complex numbers

  $$Z = \sqrt{5^2 + 4^2 = \sqrt{4} = 6{,}403}$$

- Calculate the directional angle of a complex number

  $$\theta = arc\ tg\ \frac{4}{5} = 38{,}66^0$$

- Complex numbers in polar

  $Z = (5 + j4)\Omega$

  $= M < \theta = 6{,}403\Omega < 38{,}66^0$

The examples above show how complex analysis plays an important role in solving algebraic equations involving complex numbers. Without the concept of complex numbers and complex analysis, these equations cannot be solved using ordinary algebra involving only real numbers.

The application of complex analysis in algebra is not only limited to polynomial equations and linear algebra, but also covers other areas such as number theory, group theory, and other abstract algebraic theories. With its ability to solve equations that cannot be solved with real numbers, complex analysis becomes a very important tool in the development and understanding of modern algebraic concepts.

In complex linear algebra, concepts from complex analysis are used to analyze the properties of complex matrices, such as eigenvalues and eigenvectors, as well as other problems involving operations on complex matrices (Susanti, 2021).

In group theory, complex analysis is used to study group representations through character theory. The concepts of holomorphic functions and residue theory from complex analysis play an important role in analyzing the properties of such group representations.

Overall, the application of complex analysis in solving algebraic problems is vast and covers a wide range of areas in modern algebra. With its ability to solve equations and problems that cannot be solved using only real algebra, complex analysis becomes a very important and influential tool in the development of further algebraic theories.

CONCLUSION

In this paper, we have explored strategies for applying complex analysis in solving algebraic problems. We have discussed basic concepts of complex analysis such as complex numbers, analytic functions, Laurent series, and contour integrals. Furthermore, we have presented strategies that can be used to apply complex analysis in solving algebraic problems, such as using analytic functions to simplify algebraic equations, solving algebraic equations with complex numbers, and analyzing properties of algebraic functions with complex analysis.

Through the examples presented, we have shown how the strategies can be practically applied in solving algebraic problems involving difficult integrals, algebraic equations that cannot be solved with real numbers, and analysis of the properties of algebraic functions. The application of complex analysis in solving algebraic problems opens up new opportunities to solve problems that were previously difficult to solve with ordinary algebraic methods. By utilizing the concepts of complex analysis, we can gain new insights and more effective solutions to complicated algebraic problems.

Nonetheless, the application of complex analysis in algebraic problems requires a deep understanding of the concepts of complex analysis and the ability to integrate those concepts with algebraic principles. Therefore, it is important for researchers and practitioners to continuously develop their knowledge and skills in this area.

This research certainly still has weaknesses and shortcomings. One of the weaknesses is the difficulty in obtaining sufficient references related to the strategy of applying complex analysis to solve algebraic problems. Most of the sources found focus more on discussing the basic concepts of complex analysis itself. In addition, the use of language and delivery in this paper may still not be well understood by readers in certain parts. Nevertheless, this research has contributed to formulating a comprehensive strategy for applying complex analysis to solve algebra problems. The formulated strategy includes the stages of concept identification, relevant complex analysis methods, algebraic problems to be solved, and examples of strategy application in a detailed and systematic manner. This strategy can be used as a guide in applying complex analysis to solve various algebraic problems effectively, both in academic and practical fields.

Further research can be focused on developing new strategies and methods for applying complex analysis in more complex algebraic problems, as well as exploring applications of complex analysis in other fields such as physics, engineering, and economics. Interdisciplinary collaboration can also enrich the application of complex analysis in solving problems that cross disciplinary boundaries.

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